However, now the possibility is provided of giving the charge shape entirely in advance, i.e., the quantity a_2. Then we will have the system of equations (15) and (16) to determine the constants A_0 and q. The initial data of the charge should hence satisfy the condition $\mathrm{Re}\, z(\xi) > 0$ for $q < \xi < 1$.

LITERATURE CITED

1. V. M. Kuznetsov, "On the shape of the ejection funnel during an explosion on the surface of soil," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 152-156 (1960).
2. V. M. Kuznetsov and É. B. Polyak, "Hydrodynamic-impulse scheme of computing an explosion by the ejection of cord charges," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 4, 32-39 (1973).
3. V. M. Kuznetsov, M. A. Lavrent'ev, and E. I. Sher, "On the directed hurling of soil by using an explosive," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 49-50 (1960).
4. M. A. Lavrent'ev and B. V. Shabat, Methods of Complex Variable Function Theory [in Russian], Nauka, Moscow (1973).
5. A. R. Tsitskishvili, "Application of the method of I. A. Lappo-Danilevskii to seek functions mapping a half-plane conformally on a circular polygon," Diff. Urav., 10, No. 3, 458-469 (1974).
6. R. B. Salimov, Some Fundamental Problems on the Change in Contours of the Theory of Analytic Functions and Their Application to Fluid Mechanics [in Russian], Izd. KVKIU, Kazan' (1970).
7. F. D. Gakhov, Boundary-Value Problems [in Russian], Fizmatgiz, Moscow (1963).
8. N. I. Muskhelishvili, Singular Integral Equations [in Russian], Fizmatgiz, Moscow (1962).
9. G. N. Polozhii, "Effective solution of the problem of the approximate conformal mapping of simply and doubly connected domains and determination of the Schwartz – Christoffel constants by using electrohydrodynamic analogs," Ukrainsk. Mat. Zh., 7, No. 4, 423-432 (1955).

HURLING OF SHELLS BY HOLLOW CHARGES

V. A. Odintsov, V. V. Selivanov, and S. S. Usovich

UDC 533.6.01.011

Results of the numerical solution of the problem of one-dimensional hurling of shells by hollow explosive charges are elucidated. The results of the numerical solution are compared with asymptotic formulas. Numerous domestic and foreign papers have been devoted to the question of hurling shells by explosive charges. A numerical solution of the problem of convergence of a ring to the center under the effect of detonation products is presented in [1-3]. The problem of hurling a shell by a hollow explosive charge with an internal lining is considered in [4]; the solution of the problem of hurling a shell by a hollow explosive charge without the cavity lining is presented in [5] on the basis of the energy-balance equations; however, the complete picture of the processes occurring in the detonation products is not considered.

A shell with a hollow explosive charge is shown in Fig. 1. The detonation products (DP) are initially a gas at rest with the initial density $\rho_0 = \rho_{BB}$ and the pressure $p_0 = \rho_0 D^2/8$, whose extension is described by the Landau – Stanyukovich polytropy $p = A\rho^{\kappa}$ (k = 3).

The governing parameters of the problem are the load coefficient $\beta = m/M$ and the relative cavity radius $\lambda = a_{0p}/a_0$, where m is the mass of the high-explosive charge, M is the mass of the shell, a_{0p} is the radius of the cavity in the high-explosive charge, and a_0 is the inner radius of the shell. The shell strength and compressibility are neglected. The charge is in a vacuum.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 161-166, May-June, 1976. Original article submitted June 26, 1975.

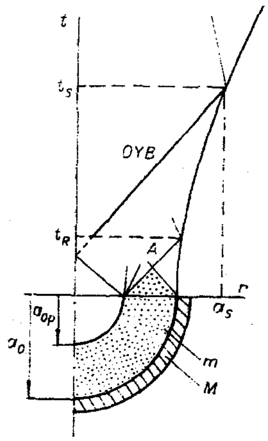

Fig. 1

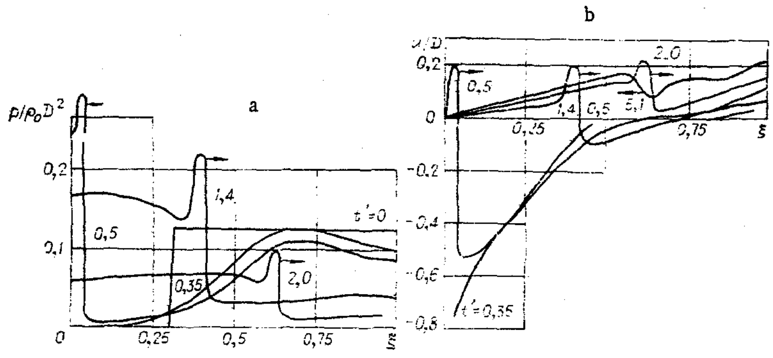

Fig. 2

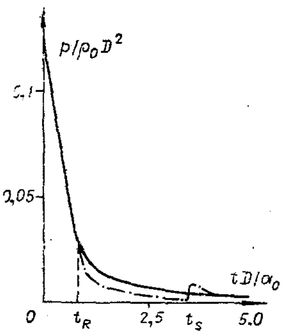

Fig. 3

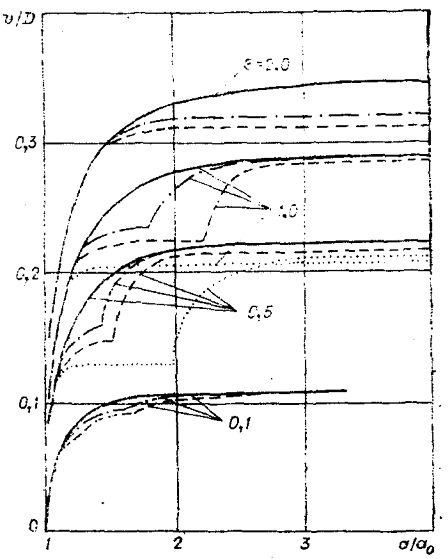

Fig. 4

An r−t diagram of the process is presented in Fig. 1. Radial expansion of the shell and escape of the gas within the cavity start at the time $t = 0$.

After the rarefaction wave fronts meet at the point A, the whole gas is completely involved in the motion. A shock forms gradually on the axis of symmetry for a high-speed spreading of the gas, which overtakes the shell at the time t_S and communicates an additional impulse to it. The line OYB is the shock-front trajectory. The gas entropy increases with the origination of the shock and its reflections from the shell and from the axis of symmetry; however, the increase in entropy is negligible $(\Delta S \sim \Delta p^3)$ in the case of reflection from the shell,

TABLE 1

λ \ β	0,1	0,5	1,0	2,0
0.3	1.16	1.43	1.80	4.50
0.5	1.20	1.53	2.23	No overtaking
0.8	1,26	1.98	4.30	No overtaking

TABLE 2

		β			
		0.1	0.5	1.0	2.0
$\frac{r_0}{D}$	Pokrovskii – Garni	0.109	0.223	0.289	0.353
	Stanyukovich	0.110	0.232	0.306	0.386
	Numerical solution	0.109	0.225	0.290	0.357

TABLE 3

		ν			
		2	4	6	8
$\frac{\rho_{max}}{\rho_{min}}$	$\lambda = 0.3$	1,68	1.52	1,45	1,27
	$\lambda = 0.5$	2.54	1.93	2.92	1,44
	$\lambda = 0.8$	4,15	2.29	3.3	1.67

TABLE 4

λ \ β_0	0.1		0.5		1,0		2,0	
	β	r_0	β	r_0	β	r_0	β	r_0
0	0,1	0,109	0,5	0,225	1.0	0,290	2,0	0,357
0.3	0,091	0,107	0,445	0,213	0,91	0,281	1,82	0,347
0,5	0,075	0,096	0,375	0,197	0,75	0.262	1.5	0,326
0,8	0,035	0,068	0.180	0,140	0,36	0,194	0,72	0.258

TABLE 5

λ \ β_0	0,1	0,5	1,	2,0
0	0,106	0,218	0,276	0.330
0.3	0.0856	0.196	—	—
0.5	0,0749	0,182	0.205	0.278
0,65	0.0599	0,153	0,167	0.238
0.8	—	—	—	0.249

and the process affects a quite small part of the gas mass when the shocks collapse on the axis of symmetry when the wave amplitude theoretically tends to infinity. Appropriate estimates are performed in [6]. This circumstance permits use of a barotropic equation of state for the detonation products in the whole flow domain.

The system of equations describing the detonation product motion is

$$\begin{gathered}\partial\rho/\partial t + \partial(\rho u)/\partial r + (\nu - 1)\rho u/r = 0;\\ \partial u/\partial t + u\partial u/\partial r + (1/\rho)\partial p/\partial r = 0,\\ p = A\rho^3,\end{gathered} \tag{1}$$

where ρ is the DP density, u is the mass flow rate of the DP, p is the pressure in the DP, r is the radial coordinate, and ν is the measure of the space ($\nu = 2$). The system (1) is integrated numerically for initial and boundary conditions.

The boundary conditions are as follows: a) on the shell $u = v$ for $r = a$, the law of incompressible thin fluid shell motion (ITF shell) is expressed in the form of Newton's law

$$M dv/dt = pS,$$

where v is the shell velocity, S is the area of the shell inner surface, p is the pressure in the shell, and a is the running inner radius of the shell; b) on the cavity boundary $p = 0$, $\rho = 0$ for $r = a_p$ (if $a_p > 0$), and $u = 0$ for $r = a_p = 0$.

Here a_p is the running radius of the cavity.

The initial conditions are

$$t = 0,\ p = p_0,\ u = 0,\ \rho = \rho_0.$$

The problem is considered in the dimensionless variables

$$\rho' = \rho/\rho_0,\ u' = u/D,\ p' = p/\rho_0 D^2,\ t' = tD/a_0,\ r' = r/a_0.$$

Integration of the system (1) is performed by a finite-difference method of the predictor–corrector type in a second-order approximation. A detailed scheme of the calculations has been presented in [6].

Calculations were performed for charges with the cavity dimensions $\lambda = 0$. The coefficient varied between the limits 0.3; 0.5; 0.8. The calculation was carried out on a BÉSM-3M electronic digital computer. The residual in the energy balance did not exceed 4%.

The pressure and velocity distributions in the DP with respect to the dimensionless coordinate $\xi = r/a$ at different times ($\beta = 0.5$, $\lambda = 0.3$) are shown in Fig. 2a and b, respectively. The front of the diverging shock is determined clearly. The radial wave motion is accompanied by a rapid drop in amplitude of the front. The shock front reflected from the shell is seen well in Fig. 2b ($t' = 5.1$).

It is most convenient to trace the singularities of the acceleration process in the presence of a cavity in a high-explosive charge by comparing the acceleration laws of a shell of fixed diameter and mass as the cavity dimension changes. In this case, the quantitities β and λ are related by the dependence $\beta = \beta_0(1 - \lambda^2)$, where β_0 is the load coefficient for a solid charge. The appropriate laws of the change in pressure on the shell ($\beta_0 = 2$, $\lambda = 0$, and $\lambda = 0.5$) are represented in Fig. 3. The pressure change law at the initial times is identical for both cases. However, later the arrival of the rarefaction wave, coming on from the inner boundary of the charge (the time t_R) and the arrival of the shock at the time t_S are felt for the charge with the cavity (dashed-dot line).

The acceleration curves $v/D = f(a/a_0)$ are represented in Fig. 4, where the solid line corresponds to a cavityless charge, the dashed-dot line, to $-\lambda = 0.3$, the dashed line to $-\lambda = 0.5$, and the points to $-\lambda = 0.8$; it is seen that for large values of λ the velocity increment because of the additional impulse of the shock can be quite significant (for $\beta = 0.5$, $\lambda = 0.5$ the velocity increment because of the effect of the shock is 32%).

A formal criterion for the end of acceleration (shutdown of the computation) is used in the form $(\Delta v/\Delta a)1/v \leq 0.05$ in computing the final velocity v_0. The relative radius a_S/a_0 at the time the shock emerges on the shell is represented in Table 1.

It is clarified that the values of the final velocities v_0/D for fixed β are practically independent of λ in some ranges of variation of β. For very large β the shell reaches its ultimate velocity earlier than the unloading wave arriving from the cavity starts to exert influence; for small values (for $\beta = 0.1$, say), the ultimate shell velocity is reached because of shock reverberations. For medium β ($0.5 < \beta < 2-3$) a situation can occur such that the shock overtakes the shell only once or generally not at all. In this case, a significant part of the initial high-explosive energy will be concentrated in the shock, which is incapable of transmitting this energy to the shell. This deduction agrees with the deductions in [5].

Values of the ultimate velocities $v_0/D = f(\beta)$ for a charge without a cavity are given in the lowest line in Table 2, where design values of the velocities v_0/D obtained by means of the asymptotic Pokrovskii–Garni (linear DP velocity distribution)

$$v_0/D = (1/2)\sqrt{\beta/(2+\beta)}$$

and Stanyukovich (parabolic DP velocity distribution)

$$v_0/D = (1/2\sqrt{2})\sqrt{3\beta/(3+\beta)}$$

are presented [4].

It follows from Table 2 that the Pokrovskii–Garni formula, based on a linear law, yields more exact agreement with the results of the numerical computation. It should be kept in mind that the density distribution hence differs substantially from the equilibrium. Shown (for $\beta = 0.1$) in Table 3 is the change in the value of the ratio ρ_{max}/ρ_{min}, which characterizes the deviation of the distribution from the equilibrium value. It is hence seen that the equilibrium stage of the process, which can be determined approximately by the condition $\rho_{max}/\rho_{min} = 1.5$, sets in relatively later for larger values of λ.

Data showing the change in the final shell velocity for fixed diameters and masses as a function of the change in cavity size are given in Table 4, where β and λ are related by the dependence $\beta = \beta_0(1-\lambda^2)$. Presented in Table 5 are values of the velocity $(v_0/D)^{1.5}$ for a fixed acceleration radius ($a'_S = 1.5$). These results physically denote cessation of shell acceleration because of rupture. It is interesting to note that for large values of β_0 the influence of the cavity radius on the final velocity $(v_0/D)^{1.5}$ is negligible (the shell is "not responsive" to the presence of a cavity and its size).

LITERATURE CITED

1. S. A. Kinelovskii, N. I. Matyushkin, and Yu. A. Trishin, "Convergence of an incompressible ring to the center under the effect of explosion products," in: Dynamics of a Continuous Medium [in Russian], No. 5, Inst. Gidrodinam., Sibirsk. Otd., Akad. Nauk SSSR, Novosibirsk (1970).
2. S. A. Kinelovskii, N. I. Matyushkin, and Yu. A. Trishin, "On the motion of a cylindrical piston towards the center," in: Dynamics of a Continuous Medium [in Russian], No. 7, Inst. Gidrodinam., Sibirsk. Otd., Akad. Nauk SSSR, Novosibirsk (1971).
3. S. A. Kinelovskii, N. I. Matyushkin, and Yu. A. Trishin, "Motion of a cylindrical piston surrounded by an expanding gas layer," in: Dynamics of a Continuous Medium [in Russian], No. 7, Inst. Gidrodinam., Sibirsk. Otd., Akad. Nauk SSSR, Novosibirsk (1971).
4. A. Tselminsh, "Methods of computing explosions in a cylindrical cavity," J. ASME, 36, Ser. E, Appl. Mech., No. 2 (1959).
5. M. Defourneaux, "Transfers d'energie dans les combustions et detonations avec confinement," Astronaut. Acta, 17 (1972).
6. A. V. Kashirskii, Yu. V. Korovin, and V. A. Odintsov, "Motion of a shell under axial detonation," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1971).